UNIT I

Q. What is Operational Research ?

Ans.

* Operational Research is a scientific method of providing executive with an analytical and objective basis for decisions. ( PMS Blackett)
* Operational Research is the art of winning war without actually fighting it. (Arthur Clarke)
* Operational Research is a scientific approach to problem solving for executive management. ( H.M. Wagner) .

Operational research (OR) encompasses a wide range of problem-solving techniques and methods applied in the pursuit of improved decision-making and efficiency, such as [simulation](http://en.wikipedia.org/wiki/Simulation), [mathematical optimization](http://en.wikipedia.org/wiki/Mathematical_optimization), [queueing theory](http://en.wikipedia.org/wiki/Queueing_theory" \o "Queueing theory) and other stochastic-process models,[Markov decision processes](http://en.wikipedia.org/wiki/Markov_Decision_Process), [econometric methods](http://en.wikipedia.org/wiki/Econometrics), [data envelopment analysis](http://en.wikipedia.org/wiki/Data_envelopment_analysis), [neural networks](http://en.wikipedia.org/wiki/Neural_networks), [expert systems](http://en.wikipedia.org/wiki/Expert_System), [decision analysis](http://en.wikipedia.org/wiki/Decision_analysis), and the [analytic hierarchy process](http://en.wikipedia.org/wiki/Analytic_hierarchy_process)

Q.History of Operational Research

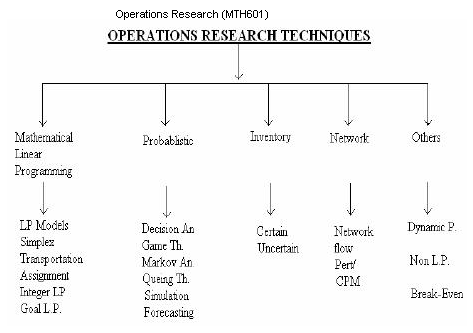
Ans. As a formal discipline, operational research originated in the efforts of military planners during World War II. In the decades after the war, the techniques began to be applied more widely to problems in business, industry and society. Since that time, operational research has expanded into a field widely used in industries ranging from petrochemicals to airlines, finance, logistics, and government, moving to a focus on the development of mathematical models that can be used to analyze and optimize complex systems, and has become an area of active academic and industrial research

In the World War II era, operational research was defined as "a scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control. Other names for it included operational analysis (UK Ministry of Defense from 1962) and quantitative management

Q.Application areas of Operational Research

Ans. Some of the industrial / government/ business problems that can be analysed by the OR approach have been arranged by functional areas as follows:

1. Finance and accounting
2. Marketing
3. Purchasing
4. Production management
5. Facilities planning
6. Manufacturing
7. Maintenance & project scheduling
8. Personnel management
9. Techniques and general management
10. Government



Constraints: There are always limitation(or constraints) on the use of resources. Such constraints must be expressed as linear equalities or inequalities in terms of decision variables. The solution of an LP must satisfy the constraint.

Q.What is inventory control. Write down the advantages and disadvantages.

Ans. Inventory is defined as any idle resources of an enterprise. It is a physical stock of goods kept for future use. In a factory the inventory may be in the form of raw material, parts, semi-finished gods. Inventory includes furniture, machinery etc.

It is essential for an enterprise to have inventory due the following resource

1. It helps to smooth an efficient running of the business
2. It provides adequate service to the customer.
3. It reduces the possibility of duplication of order
4. It helps in minimizing the loss due to the obstacle, damage etc.
5. Take advantage of price discounts by bulk purchasing
6. Items which have not been used for long time are removed from the inventory.
7. Periodic review system in which orders are placed at regular intervals of time.

Types of Inventory:

1. Lot-size or cycle inventory: it is the inventory necessary to meet the average demand during the successive replenishment. The amount of such inventory depends upon the production lot size.
2. Pipeline or Transit inventory: movement of item cannot be instantaneous, optional inventory level is required for shipment of inventory items to distribution centers and customers from production centers . such an inventory is called process inventory.
3. Safety or Buffer Inventory: it is the specific level of extra stock of inventory that is maintained for protection against uncertainties of demand and the lead time necessary for delivery of goods.
4. Seasonal Inventory : Inventory also needed for items whose sale is depend on seasonal pattern of demand and whose production is not uniform.

Advantages:

1. Economic of the production with large run size or quantity or lot.
2. Smooth and efficient running of business
3. Faster and adequate service to the customer
4. Project from the market where prices are expected to rise
5. Advantage of price discount

Disadvantages:

1. With the increase inventory, holding cost of inventory is also increases.
2. Interest or invested capital
3. Physical handling
4. Accounting
5. Depreciation and detrization.

Q. Explain Inventory costs:

Ans: The costs that are affected by the firm decision to maintain a particular level of inventory are called relevant costs. There are many types of inventory costs:

1. Item costs: It refers to the cost associated with an item whether it is manufactured or purchased. The purchase price will be considered when discount are allowed for any purchase above a certain quantity.
2. Set-up cost: These cost include the fixed cost associated with obtaining the goods through placing of an order or purchasing or manufacturing or setting up a machinery before starting the production. They include the cost of purchase, requisition, follow-up, receiving the goods, quality control etc. these are also called an order cost or replenishment cost. They are assumed to be independent of the quantity ordered or produced.
3. Carrying cost or holding cost: The cost associated with carrying or holding the goods in stock is known as holding or carrying costs. Holding costs is assumed to a directly with the size of inventory as well as the time the item is hold in stock.
4. Shortage cost or stock out cost: The penalty costs that are incurred as a result of running out of stock (i.e shortage) are known as shortage cost or stock out cost.

Q. what is demand?

Ans. Demand refers to the number of items required per period. It may be known exactly or known in terms of probability or may be completely unknown.

The demand pattern of items may be either deterministic or probabilistic. Problem in which demand is known and fixes are called deterministic problem where as those problem in which the demand is assumed to be a random variable are called stochastic demand or probabilistic demand.

Q. What is Lead time?

Ans : The time gap between the placing of an order& the actual; arrival of the inventory is known as lead time. If the lead time known ad is not equal to zero, & if the demand is deterministic, all that in requires to do is to order in advance by the tie equal to the lead time. If the lead time is zero there is no need to order in advance.

Q. What is order cycle , time horizon, recorder level?

Ans : Order cycle : the time period between placement of two successive order is referred to as an order cycle. The order cycle may be placed in the basis of the following two types of inventory review system

1. Continuously review: the record of the inventory level s checked continuously until a certain lower limit (known as recorder level) is reached when a new order is placed. This often known as two-bin system.
2. Periodic review: in this inventory levels are reviewed at equal time intervals & orders are placed at such intervals. The quantity ordered each time depends on the available inventory level at the time of review..

Time horizon: the time period over which the inventory level will be controlled is known as time horizon.

Recorder level: the level between the maxim um & minimum stock at which the purchasing of manufacturing activities must start for the replenishment is known as recorder level.

Classification of EOQ Models

List of symbols used

**C= Purchase (or Manufacturing) cost of an item (Rs per unit)**

**C0 = ordering (or set-up cost) cost per order (Rs. Per order)**

r = cost of carrying one rupee worth of inventory expressed in terms of percent of rupee value of inventory

**Ch = C\*r = cost of carrying one unit of an item in the inventory for a given length of time (Rs per item per unit time)**

Cs = Shortage cost per unit time (Rs per unit time)

**D = annual requirement (or demand) of an item**

ROL = reorder level

LT = replenishment lead time

n = number of orders per time period

t = reorder cycle time

tp = production period

rp = production rate

**TC = total inventory cost**

**TVC = total variable inventory cost**

**Question: The production department of a company requires 3,600 kg of raw material for manufacturing a particular item per year. It has been estimated that the cost of placing an order is Rs. 36 and the cost of carrying is 25 percent of the investment in the inventories. The price is Rs. 10 per kg. Help the purchase manager to determine an ordering policy of raw material**

**Question: An aircraft company uses rivets at a constant rate of 2500 per year. Each unit cost Rs 30. The company personnel estimate that it costs Rs 130 to place an order and that the carrying cost of inventory is 10 percent per year. How frequently should orders be placed? Also determine the optimum size of order**.

**Question: A manufacturer has to supply his customer with 600 units of his product per year. Shortages are not allowed and the storage cost amounts to Rs. 0.60 per unit per year. The set up cost per run is Rs. 80.00. Find the optimum run size and the minimum average yearly cost**

Question: At present a company purchases an item X from outside suppliers. The consumption of this item

is 10,000 units/year. The cost of the item is Rs 5 per unit and the ordering cost is estimated to be Rs 100

per order. The cost of carrying inventory is 25 percent. If the consumption rate is uniform, determine

the economic purchase quantity.

Question: a ) Compute EOQ and the total variable cost for the following items:

Annual demand = 500 units

Unit Price = Rs. 20

Order cost = Rs 16

Storage cost = Rs 2% per annum

Interest = 12 % per annum

Obsolescence rate = 6 % per annum

b) Determine the total variable cost that would result for the item if an

incorrect price of Rs 12.80 were used.

**Question: A purchase manager places order for a lot of 500 units of a particular item. From the available data the following results are obtained:**

**Inventory carrying cost = 40 per cent**

**Ordering cost per order = Rs 600**

**Cost per unit = Rs 50**

**Annual Demand = 1000 units**

**find out the loss to the organization due to his ordering policy**

**THEORY OF GAMES:**

**It is a decision theory applicable to competitive situations. It is helpful when two or more individuals or organizations with conflicting objectives try to make decisions. The competitors’ are referred as players. A player may be individual, a group of individuals or organization.**

**Game theory came into existence in 20th century in 1944 John Von Neumann and Oscar Morgenstern published a book name *Theory of Games and economic behavior***

**Characteristics of Game:**

1. **There are finite number if players or competitors or participants. If the number is two the fame is called two-person game, for number greater than 2 it is called n person game.**
2. **Each participant has a finite number of possible courses of action.**
3. **Each participant must know all the courses of action available to others but must not to know which of these will be chosen.**
4. **Sum of gain and losses: the sum of the game to one player is exactly is equal to the sum of losses to another player. If the sum of gain and losses is equal to zero then is called zero-sum game otherwise it is said the non-zero sum game.**

**Q. Explain the Game strategies and Payoff matrix**

**Ans. Strategy: it is a predetermined rule by which a player decides his course of action. The particular strategy by which a player optimizes his gain or losses, without knowing the competitor’s strategy is called optimal strategy. The expected outcome per play, when players follow their optimal strategy is called the value of the game. There are two types of strategy**

1. **Pure strategy: It decision rules to always select a particular course of action.**
2. **Mixed strategy : it is decision in advance of all plays to use all or some of the available courses if action in some fixed proportion. Thus a mixed strategy is a selection among pure strategies with some fixed probabilities.**

**Payoff Matrix: It is outcome of the game .Payoff (gain or game) matrix is the table showing the amounts of received by the player named at the left hand side after all possible plays of the game. The payment is made by the player named at the top of the table.**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Player B strategy** | | | |
| **Palyer A Strategy** | **B1** | **B2** | * **- - - - -** | **Bn** |
| **A1** | **a11** | **a12** |  | **a1n** |
| **A2** | **a21** | **a12** |  | **a1n** |
| **-** |  |  |  |  |
| **-** |  |  |  |  |
| **Am** | **am1** | **am2** |  | **amn** |

Q. Pure Strategies ( Minimax and Maximin Principles ) Games with Saddle Points and rules to determine saddle point.

Ans. Pure strategies : It **decision rules to always select a particular course of action.**

Maximin Principle: the minimum value in each row represents the least gain to him , if he chooses his particular strategy. These are written in matrix by row minima. Then we select the largest value among the row minimum values. The choice of player is called the maximin principle and the corresponding gain is called the maximin value of the game.

Minimax Principle: the maximum value in each column represents the maximum loss to him, if he chooses his particular strategy. These are written in the payoff matrix by column maxima. Then he will select the minimum value among the column maximum value is called the minimax principle and the corresponding loss is the minimax value of the game.

Value of the game: this is the expected payoff at the end of the fame , when each player uses his optimal strategy i.e. is the amount of payoff at an equilibrium point. A game may have more than one saddle point. A game with no saddle point is solved by choosing strategies with fixed probabilities.

Rules to determine saddle point :

1. Select the minimum (lowest) element in each row of the payoff matrix and write them under row minima. Then select largest element among these elements and enclose in a rectangle

2. Select the maximum (highest) element in each column of the payoff matrix and write then under column maxima. Then select minimum element among these elements and enclose in a circle

3. Find out the elements that is same in the circle as the well as rectangle and mark the position of such element in the matrix. This element represents the value of the game and is called the saddle point or equilibrium point.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Player B strategy** | | | |
| **Palyer A Strategy** | **B1** | **B2** | **B3** | **Row minimum** |
| **A1** | **-1** | **2** | -2  -2 | -2 |
| **A2** | **6** | **4** | - 6 | **-6** |
| **Column maximum** | **6** | **4** | -2 | **2** |

Maximin  
 Minimax value of the game

Note: the payoff amount in the saddle point position is called the value of the game.

Q. A company management and the labour union are negotiating a new three year settlement. Each of these 4 strategies

a) Hard and aggressive bargaining b) Reasoning and logical approach

c) Legalistic strategy d) Conciliatory approach

the cost to the company are given for every pair of strategy choice

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Company Strategy | | | |
| Union Strategy | I | II | III | IV |
| I | 20 | 15 | 12 | 35 |
| II | 25 | 14 | 8 | 10 |
| III | 40 | 2 | 10 | 5 |
| IV | -5 | 4 | 11 | 0 |

What strategy will the two side adopt? Also determine the value of the game

**Solution: Apply the rule of finding the saddle point**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Company Strategy | | | | Row minimum |
| Union Strategy | I | II | III | IV |  |
| I | 20 | 15 | **12** | 35 | 12 |
| II | 25 | 14 | 8 | 10 | 8 |
| III | 40 | 2 | 10 | 5 | 2 |
| IV | -5 | 4 | 11 | 0 | -5 |
| Column maximum | 40 | 15 | 12 | 35 |  |

maximin

minimax

Since Minimax= Maximin = value of the game =12 therefore the company will always adopt strategy III- Legalistic strategy and union always adopt strategy I – Hard and aggressive bargaining.

\

**heory of Queue**

**A common situation occur in everyday life is that of queuing or waiting in a line. Queue are usually seen as theaters, doctors clinic, petrol pumps, ATM counters, incoming call etc.**

**Waiting line problem arise either because**

1. **There is too much demand on the facilities so that we say that there is an excess of waiting time or inadequate number of service facilities**
2. **There is too less demand , in which case there is too much idle facility time or too many facilities**

**Structure of a queuing system: Service system**

**Arrival process Queue discipline Departure**

Service process or mechanism

Queue or waiting line

Input source or population

Serviced

customers

Balk Renege Jockey

* Customers require that services are generated at different times by a calling population, commonly known as input source.
* The manner in which customer arrives at the service facility, individually or batches at scheduled or unscheduled time is called arrival process.
* Customers form a queue is selected for service according to cetain rules known as queue discipline.
* The rate constant or random at which service is rendered is known as service process.

Calling population characteristics:

1. Size of calling population
2. Behavior of the arrivals
3. Pattern of arrival at the system

Size of calling population: There are two types

1. Finite or limited: customers arrival depend on the number of customer already in the system, the calling population is limited or finite.
2. Infinite or unlimited: if new customer’s arrival is independent of the number of customer already in the system the calling population is called infinite or unlimited. Eg. Supermarket, railway ticket window etc.

Behaviour of customer’s arrival: there are three types of behavior

1. Balk: customers do not join the queue either by seeing the number of customers already in the service system.
2. Reneging or impatient customers: Customers after joining the queue, wait or sometime in the queue but leave before being served on account of certain reasons.
3. Jockeying : customers move from one queue to another queue hoping to service more quickly
4. Patient customers: wait for service in the queue until served and does not with between waiting lines.

Pattern of arrival in the system; two types of arrival

1. Static: customers depend on the nature of arrival pattern.

The number of unscheduled arrivals to a service facility. In some fixed period of time, can be studied by a statistical probability distribution such as Poisson distribution.

1. Dynamic: it is controlled by both the customers and service facility and the customers.

Queuing process:

* The queuing process refers to the number of queues and their respective length. The number of queues single, multiple, or priority queues depend on the layout of the system.
* The length of queue depend upon the operational situations .
* The queue length in such cases depends upon the attitude of the customers.
* In some finite sources queuing systems, the maximum permissible queue is zero length.

Queue discipline: There are two types of discipline

1. Static queue discipline : If the customers are served in the order of their arrival, then this is known as **FCFS** and this is also called static queue discipline

Other discipline which is also common in used **LCFS** (last-come-first-served)

1. Dynamic queue discipline: Service in random order (**SRO**) this rule customers are selected for services at random, irrespective of their arrivals in the service system.

Priority service: this service provide by the basis of priority or urgency.

Pre-emptive priority (or emergency).

Non-Pre-emptive priority.

**Notations**:

The notations used for analyzing of a queuing system are as follows:

n= number of customers in the system (waiting and in service)

Pn  = Probability of n customers in the system

λ = average (expected) customers arrival rate or average number of customers arrivals per unit time at the place of service.

μ = average (expected) service arrival rate or average number of customers served per unit time at the place of service.

P0 = probability of no customers in the system = 1 -

s= number of service channels (service facilities or servers)

N= Maximum number of customers allowed in the system

Ls = average (expected) number of customers in the system (waiting and in service)

Lq = average (expected) number of customers in the queue (queue length)

L = average (length) of non- empty queue.

Ws= average (expected) waiting time in the system (waiting and in service)

Wq= average (expected) waiting time in the queue

Pw= probability that an arriving customer has to wait (system being busy) = 1-P0=

Classification of Queuing models:

{(a / b / c): (d /c)}

Where a = arrival distribution

b = service time (or departure) distribution

c = number of service channels (servers)

d = maximum number of customers allowed in the system (in queue plus in service)

e = queue (or service) discipline

**Single server Queuing models: {(M/M/1): (∞/FCFS)} Unlimited Queue**

**Performance Measure:**

1. **Expected number of customers in the system (customer in the line plus the customer being served)**

**Ls =**

1. **Expected number of customers waiting in the queue ;**

**Lq=**

1. **Expected waiting time for a customer in the queue**

**Wq =**

1. **Expected waiting time for a customer in the queue**

**Ws =**

1. **Variance (fluctuation) of queue length ;**

**Var (n) = λμ / (μ –λ) 2**

1. **Probability that the queue is non-empty**

**P (n>1) = (λ/μ) 2**

1. **Probability that the number of customer, n in the system exceeds a given number k ;**

**P (n>k) = (λ/μ) k+1**

1. **Expected length of non-empty queue**

**L=**

1. **Probability of an arrival during the service time when the system contains ‘r’ customers**

**P (n=r) = {λ / (μ + λ)} r {μ / (μ + λ)}**

**Question: A television repairman finds that the time spent on his jobs has an exponential distribution with a mean of 30 minutes. If he repairs the sets in the order in which they came in, and if the arrival of sets follow a Poisson distribution with an approximate average of 10 per 8 hour say, what is the repairman expected idle time for each day? How many jobs are ahead of the average set just brought in?**

Management of Replacement

The Replacement Theory in Operations Research is used in the decision making process of replacing a used equipment with a substitute; mostly a new equipment of better usage.

The replacement might be necessary due to the deteriorating property or failure or breakdown of particular equipment. The ‘Replacement Theory’ is used in the cases like; existing items have out-lived, or it may not be economical anymore to continue with them, or the items might have been destroyed either by accident or otherwise. The above discussed situations can be solved mathematically and categorised on some basis like:

1. Items that deteriorate with time e.g. machine tools, vehicles, equipments, buildings etc,
2. Items becoming out-of-date due to new developments.
3. Items which do not deteriorate but fail completely after certain amount of use like electronic parts, street lights etc.
4. The existing working staff in an organization gradually diminishing due to death, retirement, retrenchment & otherwise *(Staff Replacement)*.

Types of failure:

1. Gradual failure: it is progressive in nature. That is , as the life of an item increases, its operational efficiency also deteriorates
   1. Increased running
   2. Decrease in its productivity
   3. Decrease in the resale or salvage value.
2. Sudden failure: this type of failure occurs after some period of giving desired service rather than deterioration while in service. The period of desired services is not constant but follows some frequency distribution may be progressive, retrogressive or random in nature
   1. Progressive failure: it is the probability of failure of an item with the increase in its life.
   2. Retrogressive failure: If the probability of failure in the beginning of the life of an item is more but as time passes the chances is its failure become less.
   3. Random failure: the constant probability of failure is associated with items that fail from random causes such as physical shock not related to age.
3. Replacement of item whose efficiency deteriorates with time: when operational efficiency of an item deteriorates with time it is economical replace the same with a new one.

Q1. A firm considering the replacement of a machine, whose cost price I Rs.12,200

and its scrap value is Rs.200. From experience the running( maintanenec &

operating) costs are found to be as follows :

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Running cost | 200 | 500 | 800 | 1200 | 1800 | 2500 | 3200 | 4000 |

Soultion:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Year of service n | Running cost | Cumulative running cost | Depreciation cost | Total cost = cumulative running cost + depreciation cost) | Average cost = (total cost/year of service) |
| 1 | 200 | 200 | 12,000 | 12,200 | 12,000 |
| 2 | 500 | 700 | 12,000 | 12,700 | 6,350 |
| 3 | 800 | 1,500 | 12,000 | 13,500 | 4,500 |
| 4 | 1,200 | 2,700 | 12,000 | 14,700 | 3,675 |
| 5 | 1,800 | 4,500 | 12,,000 | 16,500 | 3,300 |
| **6** | 2,500 | 7,000 | 12,000 | 19,000 | **3,167** |
| 7 | 3,200 | 10,200 | 12,000 | 22,200 | 3,171 |
| 8 | 4,000 | 14,200 | 12,000 | 26,200 | 3,275 |

Average cost is minimum in the 6 year. Machine should be replaced in the 6 year.

Q2. The data collected in running a machine, the cost of which is Rs. 60,000 are given below

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Year | 1 | 2 | 3 | 4 | 5 |
| Resale value | 42,000 | 30,000 | 20,400 | 14,400 | 9,650 |
| Cost of spares | 4,000 | 4,270 | 4,880 | 5,700 | 6,800 |
| Cost of labour | 14,000 | 16,000 | 18,000 | 21,000 | 25,000 |

Determine the optimum period for replacement of the machine.

Solution

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Year | Running cost = cost of spare + cost of labour | Cumulative running cost | Resale value | Depreciation = 60000 – resale value | Total cost | Average cost |
| 1 | 18,000 | 18,000 | 42,000 | 18,000 | 36,000 | 36,000 |
| 2 | 20,270 | 38,270 | 30,000 | 30,000 | 68,270 | 34,135 |
| 3 | 22,880 | 61,150 | 20,400 | 39,600 | 1,00,750 | 33,583 |
| **4** | 26,700 | 87,850 | 14,400 | 45,600 | 1,33,450 | **33,368.50** |
| 5 | 31,800 | 1,19,650 | 9,650 | 50,350 | 1,70,000 | 34,000 |

Machine would be replace after 4 years.

Q3. The data on the running costs oer year abd resale price of equipment A, whose purchase price is Rs.2,00,000 are as follows;

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Running costs | 30,000 | 38,000 | 46,000 | 58,000 | 72,000 | 90,000 | 1,10,000 |
| Resale value | 1,00,000 | 50,000 | 25,000 | 12,000 | 8,000 | 8000 | 8000 |

* 1. What is the optimum period of replacement.
  2. When equipment A is two years old, equipment B, which is a new model for the same usage, is available. The optimum period for replacement is 4 years with an average cost of Rs. 72,000. Should equipment a be changes with equipment B? if so, when

**Solution:**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Year | running cost | Cumulative cost | Resale price | Depreciation = 2,00,000-resale | Total cost = cumulative cost + resale privce | Average cost = total cost / year |
| 1 | 30,000 | 30,000 | 1,00,000 | 1,00,000 | 1,30,000 | 1,30,000 |
| 2 | 38,000 | 68,000 | 50,000 | 1,50,000 | 2,18,000 | 1,09,000 |
| 3 | 46,000 | 1,14,000 | 25,000 | 1,75,000 | 2,89,000 | 96,333.33 |
| 4 | 58,000 | 1,72,000 | 12,000 | 1,88,000 | 3,60,000 | 90,000 |
| **5** | 72,000 | 2,44,000 | 8000 | 1,92,000 | 4,36,000 | **87,200** |
| 6 | 90,000 | 3,34,000 | 8000 | 1,92,000 | 5,26,000 | 87,666.66 |
| 7 | 1,10,000 | 4,44,000 | 8000 | 1,92,000 | 6,36,000 | 90,857.14 |

Average cost per year is lowest in the fifth year i.e Rs 87,200. Hence the equipment A should be replaced at the end of the fifth year.

Now find the time of replacement of equipment A by equipment B, the average cost of equipment A in the successive years is computed

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Year | running cost | Depreciation = 2,00,000-resale | Total cost = cumulative cost + resale price | Cumulative cost | Average cost = total cost / year |
| 3 | 46,000 | 50,000-25,000=25,000  25,000-12,000=13,000 | 71,000 | 71,000 | 71,000 |
| **4** | 58,000 | 4000 | 71,000 | 1,42,000 | **71,000** |
| 5 | 72,000 | - | 76,000 | 2,18,000 | 72,666.66 |
| 6 | 90,000 | - | 90,000 | 3,08,000 | 77,000 |
| 7 | 1,10,000 | - | 1,10,000 | 4,18,000 | 83,600 |

A should be replaced with equipment B when it is four year old, otherwise the average cost per year would start increasing.

**Replacement of Item that Completely Fails:**

It is difficult to predict the find particular item fail at a particular time. This uncertainty can be avoided by deriving the probability distribution of failures. Assumed that the failure occur at the end of period say‘t’ .two types of replacement policy

1. Individual replacement policy: under this policy, an item (or equipment) is replaced just after the failure in the given system. This ensures smooth running of the system.
2. Group replacement policy: sometimes breakdown of a system. The immediate replacement of the item may not be available. This may result heavy losses in such circumstances a group replacement policy can br adopted. Under this policy item are replaced
   1. Individually as and when they fail during a specified time period.
   2. In groups at the end of some suitable time period. Without waiting their failure.

Q. A computer contains 10,000 resistors. When any resistor fails, it is replaced. The cost of replacing a resistor individually is Rs. 1 only. If all the resistors are replaced at the same time, the cost per resistor would be reduced to 35 paise. The percentage of surviving resistor S(t) at the end of the month t and the probability of failure P(t) during the month t are as follows:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| T | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| S(t) | 100 | 97 | 90 | 70 | 30 | 15 | 15 |
| P(t) | - | 0.03 | 0.07 | 0.20 | 0.40 | 0.15 | 0.15 |

What is the optimal replacement plan?

**Solution:** Let N be the number of resistors replaced at the end of the month

N0 =number of resistors in the beginning = 10,000

N1=number of resistors being replaced by the end of the first month = N0P1=10,000 \* 0.03=300

N2=number of resistors being replaced by the end of the second month = N0P2 + N1P1 =10,000 \* 0.07 + 300\*030=709

N3=number of resistors being replaced by the end of the first month = N0P3+ N1P2 + N2P1

=10,000 \* 0.20 + 300\*0.7 + 709\*0.03=2,042

N4=number of resistors being replaced by the end of the first month = N0P4+N1P3+N2P2+N3P1

=10,000\*0.40 + 300\*0.20+709\*.07+2,042\*0.03 =4,171

N5=number of resistors being replaced by the end of the first month = NOP5+N1P4+N2P3+N3P2+N4P1

=10,000\*0.15+300\*0.40+709\*0.20+2042\*0.07+4171\*0.03=2030

N6=number of resistors being replaced by the end of the first month = N0P6 +N1P5+N2P4+N3P3+N4P2+N5P1

=10,000\*0.15 + 300\*0.15+709\*0.40+2042\*0.20+4171\*0.07+2030\*0.03 = 2,590

Expected life = = 1\*0.03+2\*0.07+3\*0.20+4\*0.40+5\*0.15+6\*0.15 = 4.02 month

Average number of failure per month is

= 2,488 resistors ( approx)

Total cost of individual replacement is Rs 1 per resistor will be Rs ( 2,488 \*1) = Rs 2488. The cost of replacement of all the resistors at the same time can be calculated as follows;

|  |  |  |
| --- | --- | --- |
| End of month | Total cost of Group replacement | Average cost per month |
| 1 | 300\*1+10,000\*0.35=3800 | 3,800 |
| 2 | (300+709)\*1+10,000\*0.35=4,509 | 2,254 |
| **3** | (300+709+2,042)\*1+10,000\*0.35 =6,551 | **2,183.66** |
| 4 | (300+709+2,042+4,171)+10,000\*0.35 = 10,772 | 2,680 |
| 5 | (300+709+2,042+4,171+2,030)+10,000\*0.35 = 12,752 | 2,550 |
| 6 | (300+709+2,042+4,171+2,030+2,590)+10,000\*0.35= 15,442 | 2,557 |

Optimal group replacement is after every third month because the average cost of replacement is lowest in third month.

**UNIT V**

**PERT AND CPM**

**Q. What is Project management?**

**Ans: Project: is a aggregation or collection of jobs task operation functions are activity that are related to one another in some manner and should be completed in order to achieve specific goal such as construction of building,highway construction, designing and making product**

**For example: project of foundation of building following jobs or task to be done:**

1. **Layout**
2. **Digging**
3. **Fitting side boards**
4. **Concreting**
5. **Watering and removing side boards**

**To complete the project is not the objective but we need to know how to use the available resource to get hogher level of productivity here comes the role of management. Each project has three activities**

1. **Project should be completed minimum elapse time.**
2. **Use of available resources and man power as economically as possible without delaying the project**
3. **Project should be completed with the minimum of capital investment without delaying the project**

**Project Management has three phases:**

1. **Planning :** 
   1. **It involves defining objectives of project.**
   2. **Listing of task or job that must be performed**
   3. **Determine the requirement of sources such as material, manpower etc.**
   4. **Estimate the duration and cost for the task.**
   5. **Specify inter-relationship among various tasks.**
2. **Scheduling: Once planning is over scheduling of project in which allocation of resources.**
   1. **Identify all people who will responsible for each task.**
   2. **Develop network diagram, showing the sequential interrelationship between various activities**
   3. **Based on time estimates, calculate the total time duration.**
3. **Controlling: Planning and scheduling phases of project are undertaken before the actual project starts while controlling phase is undertaken during the actual project operation. Controlling consist of reviewing the difference between schedule & actual performance. It significant differences are observed than rescheduling must be done to update & revise the remaining part of the project.**

**PERT: Project Evaluation Review Technique**

**Developed in 1956-58 by a research team to help in the planning and scheduling of the US Navy’s Polaris Nuclear submarine Missile project. The objective of the team was to efficiently paln and develops the Polaris missile system. Since 1958, this technique has proved to be useful for all jobs or projects that have as element of uncertainty in the estimation of duration**

1. **In pert three estimates are used to form a weighted average of the expected completion time of each activity**
2. **It is basically a tool for planning and control of time.**
3. **It is also called an event oriented approach**

**CPM: Critical Path Method**

**CPM method was developed by E.I. DuPont company along with Remington Rand Corporation. The aim behind its development was to provide a technique for the control of the maintenance of company’s chemical plant**

1. **In CPM , there is only one time estimate of completion time of each activity**
2. **It is used for completing of projects that involve activities of repetitive nature.**

**PERT/CPM network consists of two major component as below**

1. **Events: It is the network diagram represent project milestone such as the start or the completion of an activity (task) or activities & occurs are a particular instant of time project has been achieved. Events are commonly represented by a circle ( nodes) in the network diagram. The events can be further classified into following categories**
   1. **Merge Event: As event which represents the joint completion if more than one activity is known as merge event.**
   2. **Burst Event : An event which represents the initation of more than one activity is known as burst event**

**Events in the network diagram are identified by number. An event can be head as well as tail. An event which does not have head is called starting node pf network. An event which does not have tail is called ending event. Events can be represented by nodes.**

**Activities: Activities in network diagram represent project operation are tasks to be conducted as such as each activity except dummy consume time and resources. An arrow is commonly used to represent as activity with the heads indicating the direction of progress in the project. An arrow (i, j) between two events; the tail event’ i’ represents the start of the activity and the head event ‘j’ represents the completion of activity.**

**Activity**

Starting event completing event

Tail of the arrow Head of the arrow

Activity Activity Dummy Activity

Tail event predecessor Head event successor

Activity –Node relationship in network diagram

The activities can be further classified into the following 3 categories :

1. Predecessor Activity : an activity which must be completed before one or more other activities start is known as predecessor activity
2. Successor Activity : An activity which started immediately after one or more of other activities are completed is known as successor activity
3. Dummy activity: an activity which does not consume either any resource and/or limit is known as dummy activity.
   1. A dummy activity in the network is addes only to establish that given precedence relationship among activities of the project & is needed when.
   2. Two or more parallel activities in a project have some head and tail events.
   3. Two or more parallel activities have some of their immediate predecessor activities are common.

Q. What is network?

Ans.

1. Network analysis is the general name given to certain specific techniques which can be used for the planning, management and control of projects
2. Shows the sequential relationships among activities using nodes and arrows

Use of nodes and arrows

Arrows: An arrow leads from tail to head directionally

1) Indicate ACTIVITY, a time consuming effort that is required to perform a part of the work.

Node: A node is represented by a circle

Indicate EVENT, a point in time where one or more activities start and/or finish.

* Activity
  + A task or a certain amount of work required in the project
  + Requires time to complete
  + Represented by an arrow
* Dummy Activity
* Indicates only precedence relationships
* Does not require any time of effort
* Event
  + Signals the beginning or ending of an activity
  + Designates a point in time
  + Represented by a circle (node)
* Activity-on-node (AON) :nodes represent activities, and arrows show precedence relationships
* Activity-on-arrow (AOA);arrows represent activities and nodes are events for points in time

**Network Rules :Following rules must be followed to draw a network**

1. Initial node has only outgoing arrow that means there must be only single initial node in a network.
2. An event cannot occur until all the activities leading to it are completed.
3. An activity or event cannot occur more than once( there should be no looping in a network)
4. There must not be any other dead end accept the final node.
5. No two activity should have the same tail & same head(i.e. no parallel activity)
6. Number of arrow= No. of activity ( except dummy node)
7. Representation of network should be such that each activity must be completed to reach the each activity.
8. All constraints are interdepencies should be solved properly in network using appropriate dummy.
9. It is user practice to show the time flow from left to right. Arrows pointing in opposite direction must be avoided & as far as possible straight arrow should be used. Arrows should not crossed each other, if the crossing is not avoidable, then arrows should broken into bridge over other.

* Path
  + A connected sequence of activities leading from the starting event to the ending event
* Critical Path
  + The longest path (time); determines the project duration
* Critical Activities
  + All of the activities that make up the critical path

There are two types of computation to calculate the time estimates:

1. **Forward Pass Computation**: It takes the maximum incoming time of the node.
2. **Backward pass computation**: It takes the minimum outgoing time of the node.

There are two types of time :

1. **Earliest time** : It uses the forward pass computation
   1. Earliest start time = earliest time of TAIL event
   2. Earliest finish time = earliest start time + Given duration
2. **Latest time** : it uses the backward pass computation
   1. Latest start time = latest finish time – given duration
   2. Latest finish time = Latest time on HEAD event

**Slack OR Float or Total float:** It is the amount of time that an activity can be delayed without delaying project completion, it is calculates as the

**Slack or Float = Latest start time – earliest start time**

**OR**

**Latest finish time – earliest finish time.**

**Free Float: how much an activity completion time may be delayed without causing any delay in its**

**immediate successor activity. Calculation of free float**

**= Float – Head event of slack**

**Independent float = Free Float – Tail event of slack**

* **Critical Path**: Find the critical path is that the sequence of activities and events where there is no “slack” i.e.. Zero slack

**PERT**

* PERT is based on the assumption that an activity’s duration follows a probability distribution instead of being a single value
* Three time estimates are required to compute the parameters of an activity’s duration distribution:
  + pessimistic time (tp )(Worst) - maximum possible time, the activity will take if everything goes bad.
  + most likely time (tm )(Average) – it is the time an activity will take if executed under normal conditions.
  + optimistic time (to )(good) – shortest time required for the completion of an activity

Calulation of Expected time :

Example :

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Activity | Predecessor | Optimistic time | Most likely time | Pessimistic time |
| A | - | 4 | 6 | 8 |
| B | - | 1 | 4.5 | 5 |
| C | A | 3 | 3 | 3 |
| D | A | 4 | 5 | 6 |
| E | A | 0.5 | 1 | 1.5 |
| F | B,C | 3 | 4 | 5 |
| G | B,C | 1 | 1.5 | 5 |
| H | E,F | 5 | 6 | 7 |
| I | E,F | 2 | 5 | 8 |
| J | D,H | 2.5 | 2.75 | 4.5 |
| K | G,I | 3 | 5 | 7 |

Solution: Draw network diagram

Step 2 : calculate Expected time and variance

|  |  |  |
| --- | --- | --- |
| Activity | Expected time | Variance |
| A | 6 | 4/9 |
| B | 4 | 4/9 |
| C | 3 | 0 |
| D | 5 | 1/9 |
| E | 1 | 1/36 |
| F | 4 | 1/9 |
| G | 2 | 4/9 |
| H | 6 | 1/9 |
| I | 5 | 1 |
| J | 3 | 1/9 |
| K | 5 | 4/9 |

Step 3 Calculate Slack :

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Activity | **Duration** | Earliest time | | Latest time | | Slack |
| Start time | Finish time | Start time | Finish time |
| **A** | **6** | 0 | 6 | 0 | 6 | **0** |
| B | 4 | 0 | 4 | 5 | 9 | 5 |
| **C** | 3 | 6 | 9 | 6 | 9 | **0** |
| D | 5 | 6 | 11 | 15 | 20 | 9 |
| E | 1 | 6 | 7 | 12 | 13 | 6 |
| **F** | 4 | 9 | 13 | 9 | 13 | **0** |
| G | 2 | 9 | 11 | 16 | 18 | 7 |
| H | 6 | 13 | 19 | 14 | 20 | 1 |
| **I** | 5 | 13 | 18 | 13 | 18 | **0** |
| J | 3 | 19 | 22 | 20 | 23 | 1 |
| **K** | 5 | 18 | 23 | 18 | 23 | **0** |

Variance of path = VA + VC + VF + VI + VK =4/9 + 0 + 1/9 + 1 + 4/9 = 2

Standard deviation of path = 1.414

*z* = (24 - 23)/s = (24-23)/1.414 = .71

From the Standard Normal Distribution table: P(z < .71) = .5 + .2612 = .7612

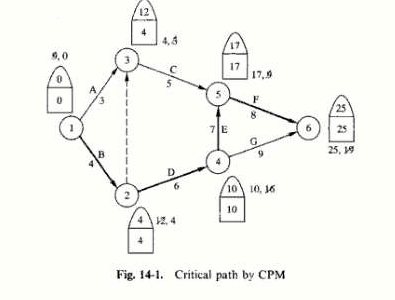
Example : Suppose the prjoject has following time activities

|  |  |  |
| --- | --- | --- |
| Activity | Predecessor | Duration |
| A | - | 3 |
| B | - | 4 |
| C | B,A | 5 |
| D | B | 6 |
| E | D | 7 |
| F | C,E | 8 |
| G | D | 9 |

1. find the critical path
2. what is the orject completion time
3. compute the independent float.

Solution :

Step1. Draw the network diagram



|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Activity | Duration | Earliest time | | Latest time | | Slack | Free float | Independent float |
| Start time | Finish time | Start time | Finish time |
| **1-2** | 4 | 0 | 4 | 0 | 4 | **0** | **0** | **0** |
| 1-3 | 3 | 0 | 3 | 9 | 12 | 9 | 1 | 1 |
| **2-4** | 6 | 4 | 10 | 4 | 10 | **0** | **0** | **0** |
| 3-5 | 5 | 4 | 9 | 12 | 17 | 8 | 8 | 0 |
| 4-5 | 7 | 10 | 17 | 10 | 17 | 0 | 0 | 0 |
| **4-6** | 9 | 10 | 19 | 16 | 25 | **6** | **6** | **6** |
| 5-6 | 8 | 17 | 25 | 17 | 25 | 0 | 0 | 0 |